Homework 4

Exercise 1

Let’s begin the exercise by modeling the data we have:

1. First model ingredient

* G: Accepting is a good idea.
* B: Accepting is a bad idea.
  + P(G)=P(B)= ½=p

1. Second Model ingredient (Payoffs)

* Ug>0, if the idea is good
* Ub<0, if the idea is bad
* Ureject=0, if we reject the idea
  + The above is because P(G)\*Ug+P(B)\*Ub=0, with

P(G)=P(B)= ½=>1/2(Ug+Ub)=0

1. Third Model Ingredient (Signals)

* High signal🡪H🡪 Suggesting a good idea.
* Low Signal🡪L 🡪 Suggesting a bad idea.
  + P(H|G)=P(L|B)=3/4=q
  + P(B|G)=P(G|B)=1/4=1-q

|  |  |  |
| --- | --- | --- |
| Signal/States | G | B |
| H | ¾ | ¼ |
| L | ¼ | ¾ |

1. We expect the payoff to be in the range of [Ug+Ub=0,Ug\*P(G|H)+Ub\*P(B|H)]

Let’s suppose that the first person gets a high signal.

P(G|H)=P(H|G)\*P(G)/P(H)= P(H|G)\*P(G)/ [P(H|G)\*P(G)+ P(H|B)\*P(B)]=

[3/4 \* 1/2]/[3/4\*1/2+1/4\*1/2]=3/4 > ½

As a result, the expected payoff shifts from 0 to a

positive number, and so they should accept the option. So if the signal is high they’ll accept the option.

If the signal is low:

P(G|L)=P(L|G)\*P(G)/P(L)= P(L|G)\*P(G)/ [P(L|G)\*P(G)+ P(L|B)\*P(B)]=

[1/4 \* 1/2]/[1/4\*1/2+3/4\*1/2]=1/4 < ½ .

So if the signal is low, they’ll reject the option.

1. Let’ s say that we have a sequence of signals. We have:

P(G|S)=P(S|G)\*P(G)/( P(S|G)\*P(G)+ P(S|B)\*P(B)) and if we expand…

p\*qa \*(1-q)b/ [p\*qa \*(1 − q)b + (1 − p)\*(1 − q)a \*qb ], with a=# high signals

b =# low signals

So if we have H,H 🡪 a=2, b=0 => … (1/2\*(3/4)2 \*1)/[ 1/2\*(3/4)2 \*1+1/2\*(1/4)2 \*1]=

=[9/32]/[10/32]=**0.9**>1/2 So he’ll definitely choose Accept, which is obvious

H,L 🡪 a=1, b=1 => .. 1/2\*(3/4)\*(1/4)/[ 1/2\*(3/4)\*(1/4)+1/2\*(1/4)\*(3/4)]=**1/2=p**

And the same is for L,H. That means this is more of a coin flip decision and it is considered a tie.

L,L 🡪 a=0, b=2 => (1/2) \* (1) \*(1/4)2 /[(1/2) \* (1) \*(1/4)2 + (1/2)\*(1)\*(3/4)2]=

=(1/32)/[10/32]=**0.1**, which he’ll clearly reject the option.

1. If the first 2 people have a tie the third person that will get the next signal will be used as a tie breaker and there won’t be any chance of a reject/ accept cascade. The only chances of this happening is H,H and L,L and then a different signal.

P(G|H,H,L)=P(H,H,L|G)\*P(G)/P(H,H,L)=

=P(H|G)P(H|G)P(L|G)\*P(G)/[ P(H|G)P(H|G)P(L|G)\*P(G)+ P(H|B)P(H|B)P(L|B)\*P(B)]=

=[3/4\*3/4\*1/4\*1/2]/[ 3/4\*3/4\*1/4\*1/2+1/4\*1/4\*3/4\*1/2]=**9/12**>1/2 so no matter the signal of third we’ll have a Accept cascade.

Moreover for L,L,H we have:

P(G|L,L,H)=P(L,L,H|G)\*P(G)/P(L,L,H)=

=P(L|G)P(L|G)P(H|G)\*P(G)/[P(L|G)P(L|G)P(H|G)\*P(G)+ P(L|B)P(L|B)P(H|B)\*P(B)]=

=[1/4\*1/4\*3/4\*1/2]/[ 1/4\*1/4\*3/4\*1/2+3/4\*3/4\*1/4\*1/2]=**3/12**<1/2 so we’ll have a reject cascade for now on.

Exercise 2

1. Let’s say that the first set of nodes is 6,7 and 11 with threshold q=1/2.
   * ***1st step:***  From node 6, we have nodes 1,5 and 10. We have ½ of their neighbors as B which is >= ½ of A. So they switch. The same thing happens for node 7 and nodes 3,8 and 12. Node 11 also switches 13,14,15 and node 2 also changes cause all of its neighbors are B.
   * ***2nd step:*** Nodes 4,9 and 16 now change cause all of their neighbors are now changed.
2. If we pick nodes 4, 9 and 16 we have:
   * ***1st step:*** Nodes 1, 5, 10 have ½ of their neighbors changed so they change. Also nodes 3, 8 ,12 change cause of node 9 is one of their 2 neighbors. Nodes 13,14, 15 also change.
   * ***2nd step:*** Nodes 6, 7 and 11 now change cause 3/5 neighbors change to B. A percentage larger than the ½ needed for a change.
   * ***3rd step.*** Node 2 now changes.
3. A cluster of density p is a set of nodes such that each node in the set has at least a p fraction of its network neighbors in the set. In this network we have clusters [1,4,5,6,10], [3,7,8,9,12], [11,13,14,15,16] which all have density=3/5 cause of nodes 6, 7 and 8.
4. The above explains why 2 nodes can’t create a cascade. If we start from any node on either of the sets described on a and b there is always one more cluster with density > 1/2 which from theory can’t cascade the information.

Exercise 3

1. We use the ICM algorithm from node 5 at t=0.
   1. t=1 🡪 Receiver nodes: 2, 4 and 1. We check (i-j)mod3

Node 1🡪 (5-1)mod3=4mod3=1. So it activates.

Node 4🡪 (5-4)mod3=1mod3=1. So it activates.

Node 2🡪 (5-2)mod3=3mod3=0. It doesn’t activate.

* 1. t=2 🡪 Receiver nodes from 1: 6.

Node 6 🡪 (1-6)mod3=-5mod3=(2\*3-5)mod3=1mod3=1. So it activates.

Receiver nodes from 4: 2.

Node 2 🡪 (4-2)mod3=2mod3=2. Not activated.

* 1. t=3 🡪 Receiver nodes from 6: 4,5.

Node 4 🡪 6-4mod3=2mod3=2. Not activated.

Node5 🡪 Already activated.

So the activated nodes are 1,4,5,6.

Exercise 4

In the above example this graph could give a full cascade if the light orange node (if the activation threshold was <=1/2) was activated with \* node as seeds. Using the greedy algorithm though, we have the orange nodes as seeds (or any other node in the left cluster) and the cascade of info in two steps will stop in the grey nodes.